

# Mathematics Subject Matter Requirements

## Part I: Content Domains for Subject Matter Understanding and Skill in Mathematics

### Domain 1. Algebra

Candidates demonstrate an understanding of the foundations of the algebra contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of algebra and its underlying structures, candidates have a deep conceptual knowledge. They are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations. They understand the power of mathematical abstraction and symbolism.

#### 1.1 Algebraic Structures

- a. Know why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings)
- b. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if  $a < b$  and  $c < 0$ , then  $ac > bc$ )
- c. Know that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 1.0, 2.0; Grade 7, Algebra and Functions: 1.0; Algebra I: 1.0, 3.0-7.0, 9.0-15.0, 24.0, 25.0; Geometry: 1.0, 17.0; Algebra II: 1.0-8.0, 11.0, 24.0, 25.0; Trigonometry: 17.0; Mathematical Analysis: 2.0; Linear Algebra: 9.0, 11.0)

#### 1.2 Polynomial Equations and Inequalities

- a. Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming)
- b. Prove and use the following:
  - ♦ The Rational Root Theorem for polynomials with integer coefficients
  - ♦ The Factor Theorem
  - ♦ The Conjugate Roots Theorem for polynomial equations with real coefficients
  - ♦ The Quadratic Formula for real and complex quadratic polynomials
  - ♦ The Binomial Theorem
- c. Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra

(Mathematics Content Standards for California Public Schools, Grade 7, Algebra and Functions: 2.0-4.0; Algebra I: 1.0, 2.0, 4.0-10.0, 12.0-15.0, 17.0-23.0; Algebra II: 2.0-11.0, 16.0, 17.0; Trigonometry: 17.0, 18.0; Mathematical Analysis: 4.0, 6.0)

### 1.3 Functions

- a. Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions)
- b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)
- c. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems)

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 1.0; Grade 7, Number Sense: 1.0, 2.0; Algebra and Functions: 3.0; Algebra I: 3.0-6.0, 10.0, 13.0, 15.0-18.0, 21.0-23.0; Algebra II: 1.0-4.0, 6.0-17.0, 24.0, 25.0; Trigonometry: 2.0, 4.0-8.0, 19.0; Mathematical Analysis: 6.0, 7.0; Calculus: 9.0)

### 1.4 Linear Algebra

- a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples and scalar (dot) and cross products
- b. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product)
- c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations)

(Mathematics Content Standards for California Public Schools, Algebra I: 9.0; Algebra II: 2.0; Mathematical Analysis: 1.0; Linear Algebra: 1.0-12.0)

## Domain 2. Geometry

Candidates demonstrate an understanding of the foundations of the geometry contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of geometry and its underlying structures, candidates have a deep conceptual knowledge. They demonstrate an understanding of axiomatic systems and different forms of logical arguments. Candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry, including coordinate, synthetic, non-Euclidean, and transformational geometry.

### 2.1 Parallelism

- a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)
- b. Know that variants of the Parallel Postulate produce non-Euclidean geometries (e.g., spherical, hyperbolic)

(Mathematics Content Standards for California Public Schools, Algebra I: 8.0, 24.0; Geometry: 1.0-3.0, 7.0, 13.0)

### 2.2 Plane Euclidean Geometry

- a. Prove theorems and solve problems involving similarity and congruence
- b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse)

- c. Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)
- d. Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8)
- e. Use techniques in coordinate geometry to prove geometric theorems

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 2.0, 3.0; Measurement and Geometry: 2.0; Grade 7, Measurement and Geometry: 1.0-3.0; Algebra I: 8.0, 24.0; Geometry: 1.0-6.0, 8.0-16.0, 18.0-21.0; Algebra II: 16.0, 17.0; Trigonometry: 12.0-14.0, 18.0, 19.0; Mathematical Analysis: 5.0)

### 2.3 Three-Dimensional Geometry

- a. Demonstrate an understanding of parallelism and perpendicularity of lines and planes in three dimensions
- b. Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres)

(Mathematics Content Standards for California Public Schools, Grade 6, Measurement and Geometry: 1.0; Grade 7, Measurement and Geometry: 2.0; Algebra I: 24.0; Geometry: 2.0, 3.0, 12.0, 17.0; Mathematical Analysis: 5.0)

### 2.4 Transformational Geometry

- a. Demonstrate an understanding of the basic properties of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection)
- b. Understand and prove the basic properties of dilations (e.g., similarity transformations or change of scale)

(Mathematics Content Standards for California Public Schools, Geometry: 11.0, 22.0)

## Domain 3. Number Theory

Candidates demonstrate an understanding of the number theory and a command of the number sense contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of number theory and its underlying structures, candidates have a deep conceptual knowledge. They prove and use properties of natural numbers. They formulate conjectures about the natural numbers using inductive reasoning, and verify conjectures with proofs.

### 3.1 Natural Numbers

- a. Prove and use basic properties of natural numbers (e.g., properties of divisibility)
- b. Use the Principle of Mathematical Induction to prove results in number theory
- c. Know and apply the Euclidean Algorithm

- d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational)

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 2.0; Grade 7, Number Sense: 1.0; Algebra I: 1.0, 2.0, 12.0, 24.0, 25.0; Geometry: 1.0; Algebra II: 21.0, 23.0, 25.0; Mathematical Analysis: 3.0)

#### **Domain 4. Probability and Statistics**

Candidates demonstrate an understanding of the statistics and probability distributions for advanced placement statistics contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of probability and statistics and their underlying structures, candidates have a deep conceptual knowledge. They solve problems and make inferences using statistics and probability distributions.

##### **4.1 Probability**

- a. Prove and apply basic principles of permutations and combinations
- b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles)
- c. Use and explain the concept of conditional probability
- d. Interpret the probability of an outcome
- e. Use normal, binomial, and exponential distributions to solve and interpret probability problems

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 3.0; Algebra II: 18.0-20.0; Probability and Statistics: 1.0-4.0; Advanced Probability and Statistics: 1.0-4.0, 7.0, 9.0, 17.0, 18.0)

##### **4.2 Statistics**

- a. Compute and interpret the mean, median, and mode of both discrete and continuous distributions
- b. Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions
- c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results
- d. Know the method of least squares and apply it to linear regression and correlation
- e. Know and apply the chi-square test

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 1.0, 2.0; Grade 7, Statistics, Data Analysis, and Probability: 1.0; Probability and Statistics: 5.0-7.0; Advanced Probability and Statistics: 4.0-6.0, 8.0, 10.0-13.0, 15.0-17.0, 19.0)

## Domain 5. Calculus

Candidates demonstrate an understanding of the trigonometry and calculus contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of trigonometry and calculus and their underlying structures, candidates have a deep conceptual knowledge. They apply the concepts of trigonometry and calculus to solving problems in real-world situations.

### 5.1 Trigonometry

- Prove that the Pythagorean Theorem is equivalent to the trigonometric identity  $\sin^2x + \cos^2x = 1$  and that this identity leads to  $1 + \tan^2x = \sec^2x$  and  $1 + \cot^2x = \csc^2x$
- Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle)
- Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems)
- Know and apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan)
- Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem)

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0, 14.0, 18.0, 19.0; Algebra II: 24.0, 25.0; Trigonometry: 1.0-6.0, 8.0-11.0, 19.0; Mathematical Analysis: 1.0, 2.0; Calculus: 18.0, 20.0)

### 5.2 Limits and Continuity

- Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit
- Show that a polynomial function is continuous at a point
- Know and apply the Intermediate Value Theorem, using the geometric implications of continuity

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0; Algebra II: 1.0, 15.0; Mathematical Analysis: 8.0; Calculus: 1.0-4.0)

### 5.3 Derivatives and Applications

- Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative
- Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change)
- Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's Theorem, the Mean Value Theorem, and L'Hopital's rule
- Use the derivative to solve rectilinear motion, related rate, and optimization problems
- Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity)
- Solve separable first-order differential equations and apply them to growth and decay problems

(Mathematics Content Standards for California Public Schools, Algebra I: 5.0-8.0, 10.0, 11.0, 13.0, 21.0, 23.0; Geometry: 3.0; Algebra II: 1.0, 9.0, 10.0, 12.0, 15.0; Trigonometry: 7.0, 15.0-19.0; Mathematical Analysis: 5.0, 7.0; Calculus: 1.0, 4.0-12.0, 27.0)

#### **5.4 Integrals and Applications**

- a. Derive definite integrals of standard algebraic functions using the formal definition of integral
- b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums)
- c. Prove the Fundamental Theorem of Calculus, and use it to interpret definite integrals as antiderivatives
- d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 9.0; Calculus: 13.0-23.0)

#### **5.5 Sequences and Series**

- a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number)
- b. Determine convergence of a given sequence or series using standard techniques (e.g., Ratio, Comparison, Integral Tests)
- c. Calculate Taylor series and Taylor polynomials of basic functions

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0, 25.0; Algebra II: 21.0-23.0; Mathematical Analysis: 8.0; Calculus: 23.0-26.0)

### **Domain 6. History of Mathematics**

Candidates understand the chronological and topical development of mathematics and the contributions of historical figures of various times and cultures. Candidates know important mathematical discoveries and their impact on human society and thought. These discoveries form a historical context for the content contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999; e.g., numeration systems, algebra, geometry, calculus).

#### **6.1 Chronological and Topical Development of Mathematics**

- a. Demonstrate understanding of the development of mathematics, its cultural connections, and its contributions to society
- b. Demonstrate understanding of the historical development of mathematics, including the contributions of diverse populations as determined by race, ethnicity, culture, geography, and gender

## **Part II: Subject Matter Skills and Abilities Applicable to the Content Domains in Mathematics**

Candidates for Single Subject Teaching Credentials in mathematics use inductive and deductive reasoning to develop, analyze, draw conclusions, and validate conjectures and arguments. As they reason, they use counterexamples, construct proofs using contradictions, and create multiple representations of the same concept. They know the interconnections among mathematical ideas, and use techniques and concepts from different domains and sub-domains to model the same problem. They explain mathematical interconnections with other disciplines. They are able to communicate their mathematical thinking clearly and coherently to others, orally, graphically, and in writing, through the use of precise language and symbols.

Candidates solve routine and complex problems by drawing from a variety of strategies while demonstrating an attitude of persistence and reflection in their approaches. They analyze problems through pattern recognition and the use of analogies. They formulate and prove conjectures, and test conclusions for reasonableness and accuracy. They use counterexamples to disprove conjectures.

Candidates select and use different representational systems (e.g., coordinates, graphs). They understand the usefulness of transformations and symmetry to help analyze and simplify problems. They make mathematical models to analyze mathematical structures in real contexts. They use spatial reasoning to model and solve problems that cross disciplines.

(Mathematics Content Standards for California Public Schools, Grade 6, Mathematical Reasoning: 1.0-3.0; Grade 7, Mathematical Reasoning: 1.0-3.0)